Neutrino masses and mixing: Singular mass matrices and Quark-lepton symmetry

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Abstract

We suggest an approach to explain the observed pattern of the neutrino masses and mixing which employs the weakly violated quark-lepton equality and does not require introduction of an *ad hoc* symmetry of the neutrino sector. The mass matrices are nearly equal for all quarks and leptons. They have very small determinant and hierarchical form with expansion parameter $\lambda \sim \sin\theta_c \sim \sqrt{m_\mu/m_\tau}$. The latter can be realized, e.g., in the model with $U(1)$ family symmetry. The equality is violated at the $\sim \lambda^2$ level. Large lepton mixing appears as a result of summation of the neutrino and charged lepton rotations which diagonalize corresponding mass matrices in contrast with the quark sector where the up quark and down quark rotations cancel each other. We show that the flip of the sign of rotation in the neutrino sector is a result of the seesaw mechanism which also enhances the neutrino mixing. In this approach one expects, in general, deviation of the 2-3 mixing from maximal, $s_{13} \sim (1-3)\lambda^2$, hierarchical neutrino mass spectrum, and $m_{ee} < 10^{-2}$ eV. The scenario is consistent with the thermal leptogenesis and (in SUSY context) bounds on lepton number violating processes, like $\mu \to e\gamma$.

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I. INTRODUCTION

One of the main results in the neutrino physics is a surprising pattern of the lepton mixing which differs substantially from the quark mixing pattern. The 2-3 leptonic mixing is maximal or nearly maximal, the 1-2 mixing is large but not maximal and the 1-3 mixing is small or very small (see [1, 2] for recent reviews). No apparent regularities or relations between mixing parameters as well between mass ratios of different fermions have been found, except for probably accidental relation $\theta_{12} + \theta_C = \theta_{23} \sim 45°$. Furthermore, the data on masses and mixings show some degree of "chaoticity".

In this connection, there are two essential issues on the way to the underlying physics:

- Quark-lepton symmetry: Is it still realized at some level? and

- New symmetry of Nature behind neutrino masses and mixings: Does it exist?

As is well known, the exact quark-lepton symmetry is violated by difference of masses of quarks and charged leptons of the first and second generations. It seems that neutrino mixing further deepens this difference.

On the other hand, several features of the neutrino data indicate certain symmetry which is not realized in the quark and charged lepton sectors (we will call it the "neutrino symmetry"):

- Maximal (or near maximal) 2-3 mixing;

- Small 1-3 mixing: the fact that

$$\sin\theta_{13} \ll \sin\theta_{12} \times \sin\theta_{23} \tag{1}$$

indicates some special structure of the mass matrix;

- Possible quasi-degenerate neutrino mass spectrum. This is hinted by (i) a general consideration in physics according to which large mixing is associated with degeneracy, ii) the neutrinoless beta decay result [3, 4, 5, 6, 7], (iii) the cosmological analysis which uses particular set of observations [8] (see however [9, 10, 11]).

These features can be related. The same symmetry can lead to the maximal 2-3 mixing and zero 1-3 mixing. So, breaking of the symmetry will generate simultaneously the non-zero 1-3 mixing and deviation of the 2-3 mixing from maximal value. (See however

[12].) Maximal mixing can be related to the quasi-degenerate mass spectrum, *etc.*

There is a number of studies which explore various "neutrino symmetries" like Z_2, A_4 [13] or $SO(3)$ [14, 15, 16] (see [17] for review). Apparently these symmetries being exact or approximate cannot be extended to the charged lepton sector where the hierarchy of masses, and in particular, inequality $m_\mu \ll m_\tau$, exists. Even more difficult is to include in the same scheme quarks which show small mixings. Realization of "neutrino symmetries" usually requires introduction of (i) new leptons and quarks, (ii) complicated Higgs sector to break the symmetry, (iii) additional symmetries to forbid unwanted couplings associated to new fermions and scalars, *etc.* Thus, in the "neutrino symmetry" scenario the observed pattern of mixing has profound implications and requires substantial extensions of known structures.

It is not excluded, however, that the "neutrino symmetry" is just misleading interpretation. In fact, till now the only solid indication of the new symmetry is the maximal 2-3 mixing. Notice, however that $\sin^2 2\theta_{23} = 1$ is obtained as the best fit point in the 2ν analysis of the atmospheric neutrino data [18]. At 90 % C.L. $\sin^2 2\theta_{23} > 0.9$ [18]. The K2K experiment gives even weaker bound on 2-3 mixing [19]. Furthermore, $\sin^2 2\theta_{23}$ is a bad quantity to describe the deviation of mixing from maximal. From theoretical point of view the relevant parameter would be

$$D_{23} \equiv 1/2 - \sin^2 \theta_{23}. \tag{2}$$

Then the present experimental bound on the deviation is

$$|D_{23}| < 0.15 \quad (90\%\text{C.L.}). \tag{3}$$

That is, $|D_{23}| \sim \sin^2 \theta_{23}$ is still possible and at the moment we cannot say that the 2-3 mixing is really near maximal one. Moreover, the latest analysis, of the atmospheric neutrino data (without renormalization of the original fluxes) shows some excess of the *e*-like events at low energies (the sub-GeV events) and the absence of excess in the multi-GeV sample. This gives a hint of non-zero D_{23} [18]. The deviation can show up in the generic 3ν analysis of the data with the solar oscillation parameters taken into account.

In this connection we will explore an opposite "no-neutrino symmetry" approach which

does not rely on a special symmetry for the neutrino sector. In contrast, we will employ the quark-lepton symmetry as much as possible.

Some elements of our approach have already been considered before.

We use the mass matrix structure which leads to mixing angles of the order

$$\tan\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}, \tag{4}$$

where m_i are the eigenvalues [20, 21, 22].

The enhancement of lepton mixing is a result of summation of rotations which diagonalize the neutrino and charged lepton mass matrices [23]. In contrast, the rotations cancel each other in the quark sector thus leading to small quark mixing. In this case the atmospheric mixing angle equals

$$\theta_{23} \sim \sqrt{\frac{m_2}{m_3}} + \sqrt{\frac{m_\mu}{m_\tau}}. \tag{5}$$

The ratio of neutrino masses is bounded from below by mass squared differences measured in the solar (Δm_{12}^2) and the atmospheric (Δm_{23}^2) neutrino experiments:

$$\frac{m_2}{m_3} \geq \sqrt{\frac{\Delta m_{12}^2}{\Delta m_{23}^2}} = 0.18^{+0.22}_{-0.08}. \tag{6}$$

The corresponding mass ratio for the charged leptons is smaller: $m_\mu/m_\tau \approx 0.06$. Even taking equality in (6) (which would correspond to the hierarchical mass spectrum) we find $\theta_{23} \sim 38^0$ which is well within the allowed region.

We employ the seesaw mechanism [24] and partial seesaw enhancement of the neutrino mixing [25].

We also posit a symmetric form for the mass matrix structure. It is in the case of symmetric matrices that the strong mass hierarchy and large mixing can be reconciled provided that the determinant of matrix is very small.

Finally, in the "democratic approach" the idea that to leading approximation all the mass matrices in the lepton sector are proportional to each other has been pursued in [26]. It has been further extended to the quark sector as well in [27].

The paper is organized as follows. In Section II we formulate our "no-neutrino symmetry" approach. In Section III we describe main features of the mass matrices and find masses and mixing angles. In Section IV we obtain generic predictions of the approach. In Section V we

consider the theoretical implications. Conclusions follow in Section VI. Numerical results are presented in the Appendix.

II.   NO-NEUTRINO SYMMETRY APPROACH

In what follows we assume the following.

1). The weakly broken quark-lepton symmetry is realized in terms of the mass matrices and not in terms of observables (masses and mixing angles). The Yukawa couplings for all quarks and leptons are nearly equal, so that the matrices of the couplings can be written as

$$\hat{Y}_K \approx \hat{Y}_0 + \delta\hat{Y}_K, \quad K = u, d, l, D. \tag{7}$$

Here index D refers to the Dirac type matrix of neutrinos. The dominant structure is given by $\hat{Y}_0$ which is common for all fermions, whereas the matrices of small corrections, $\delta\hat{Y}_K$, are different for different fermions. The smallness of $\delta\hat{Y}_K$ can be specified in two different ways which have different theoretical implications:

$$(\delta\hat{Y}_K)_{ij} \ll (\hat{Y}_0)_{ij}, \tag{8}$$

that is, the relative corrections are small to all matrix elements, or

$$(\delta\hat{Y}_K)_{ij} \ll 1, \tag{9}$$

if the largest element, $(\hat{Y}_0)_{33}$, is normalized to 1. In what follows for definiteness we will elaborate on the first possibility.

2). We assume that the matrix $\hat{Y}_0$ is singular: whole matrix $\hat{Y}_0$ as well as the sub-matrices 2-3 and 1-3 have zero (very small) determinants. As a consequence, $\hat{Y}_0$ is "unstable" in a sense that small perturbations, $\delta\hat{Y}_K$, lead to significant difference in the eigenvalues (masses) and eigenstates (mixings). This allows us to explain (see Section III) substantial deviation from the quark-lepton symmetry at the level of observables.

In what follows we consider the following symmetric singular structure

$$\hat{Y}_0 = \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda \\ \lambda^2 & \lambda & 1 \end{pmatrix}, \tag{10}$$

where the expansion parameter

$$\lambda \sim \sin\theta_c \sim 0.2 - 0.3. \tag{11}$$

We will comment on other possibilities in Section V.

3). The smallness of neutrino mass is explained by the seesaw mechanism [24]:

$$\hat{m}_\nu = -\hat{m}_D \hat{M}_R^{-1} \hat{m}_D^T \quad , \tag{12}$$

where $\hat{m}_D \equiv \hat{Y}_D v_1$ is the Dirac mass matrix, and v_1 is the electroweak vacuum expectation value (VEV) which generates masses of the upper fermions. The seesaw type II contribution, if exists, is small and can contribute to the correction matrix $\delta\hat{Y}_K$.

For simplicity we assume that mass matrix of the right-handed (RH) neutrinos, $\hat{M}_R$, has the same structure as given in Eqs. (7) and (10). This could correspond to a situation when all fermionic components are in the same multiplet and the flavor information is in fermions, whereas Higgs multiplets are flavorless. In general, this is not necessary, since the RH neutrino mass matrix has different gauge properties and is generated by different Higgs multiplet VEV. Also it may have different expansion parameter.

The seesaw mechanism plays the triple role here. (i) It explains smallness of the neutrino mass. (ii) It flips the sign of rotation which diagonalizes the light neutrino mass matrix, so that in the lepton sector the up and down rotations sum up (in contrast to the quark sector) thus leading to large lepton mixing. (iii) It enhances moderately (by factor of ~ 2) the mixing angles which come from the neutrino mass matrix. The last two facts—flipping of the relative sign of rotations and the moderate seesaw enhancement of the neutrino mixing angle—lead to large lepton mixings.

The situation is different in the quark sector. The same dominant form for the mass matrices of the up and down quarks leads due to cancellation of rotations to zero mixing equal to the identity matrix. The CKM matrix originates from the mismatch between correction matrices $\delta\hat{Y}_u$ and $\delta\hat{Y}_d$ which appear small in our approach. This in turn guarantees the smallness of the CKM angles.

We parametrize the complete matrix of Yukawa coupling (7) as

$$\hat{Y}_K = \begin{pmatrix} (1+\epsilon_{11}^K)\lambda^4 & (1+\epsilon_{12}^K)\lambda^3 & (1+\epsilon_{13}^K)\lambda^2 \\ (1+\epsilon_{12}^K)\lambda^3 & (1+\epsilon_{22}^K)\lambda^2 & (1+\epsilon_{23}^K)\lambda \\ (1+\epsilon_{13}^K)\lambda^2 & (1+\epsilon_{23}^K)\lambda & 1 \end{pmatrix} y_K, \tag{13}$$

where the range for the corrections, ϵ_{ij}, is restricted by λ:

$$|\epsilon_{ij}^K| \leq \lambda, \quad K = u, d, l, D, M, \tag{14}$$

for all i, j in the first case (8). The overall multipliers, $y_K \simeq 1$, describe the amount of non-unification of the third generation of quarks and leptons. They can also be introduced as the corrections to 33 elements: $1 \to (1 + \epsilon_{33}^K)$.

The mass matrices (without renormalization group effects) equal:

$$\begin{aligned} \hat{m}_K &= \hat{Y}_K v_1, & K &= u, D, \\ \hat{m}_K &= \hat{Y}_K v_2, & K &= d, l, \\ \hat{M}_R &= \hat{Y}_K M_0, & K &= M. \end{aligned} \tag{15}$$

Here v_1 and v_2 are the VEVs of the two Higgs doublets and M_0 is the overall scale of RH neutrino masses.

In what follows we will consider for simplicity ϵ_{ij} to be real.

III. SINGULAR MASS MATRICES, MASSES AND MIXINGS

A. Expansion parameter

The value of expansion parameter is determined essentially by the condition (14) which encodes degree of violation of the quark-lepton symmetry in our approach and by the ratio of muon to tau lepton masses which shows the weakest mass hierarchy. According to (13) we obtain

$$\frac{m_\mu}{m_\tau} \approx \lambda^2(\epsilon_{22}^l - 2\epsilon_{23}^l) \sim \lambda^2 \cdot \epsilon \leq \lambda^3, \tag{16}$$

where, in general, by ϵ we will denote combinations of ϵ_{ij} of the order ϵ_{ij}.

There are two solutions of Eq. (16) depending on the sign of the mass ratio. If $m_\mu/m_\tau < 0$, the smallest value of λ would correspond to $\epsilon_{22}^l \sim -\lambda$ and $\epsilon_{23}^l = \lambda$, so that the ratio equals

$3\lambda^3$, and consequently,

$$\lambda \sim \left(-\frac{m_\mu}{3m_\tau}\right)^{1/3}. \tag{17}$$

Using value of the mass ratio at the GUT scale, $m_\mu/m_\tau = 0.045$, we obtain $\lambda \geq 0.26$. In this case the corrections enhance the mixing:

$$\tan 2\theta_{23}^l = \frac{2(1+\epsilon_{23}^l)\lambda}{1 - (1+\epsilon_{22}^l)\lambda^2} \approx \frac{2(1+\lambda)\lambda}{1-\lambda^2}. \tag{18}$$

For $m_\mu/m_\tau > 0$ the smallest λ corresponds to $\epsilon_{22}^l \sim \lambda$ and $\epsilon_{23}^l = -\lambda$. The required value of λ is approximately the same but the mixing is smaller.

Notice that $\lambda = \sin\theta_c = 0.22$ would require $\epsilon_{22}^l - 2\epsilon_{23}^l = 1 - 2$, that is, large corrections.

Stronger mass hierarchy of quarks can be obtained taking values of ϵ_{22} and ϵ_{23} closer to zero. In Fig. 1 we show the lines of constant mass ratios m_2/m_3 in the ϵ_{22}-ϵ_{23} plane for the quarks and charged leptons. The figure indicates certain hierarchy of the 22 and 23 corrections: $\epsilon^u \ll \epsilon^d \ll \epsilon^l$. However, this hierarchy cannot be established for all matrix elements due to the need to reproduce observed mixing angles. In particular, value of the 2-3 CKM mixing still prevents the deviations in the 2-3 sector of the up and down quarks from being extremely small simultaneously.

Explanation of other observables, especially in the 1-2 sectors, requires that some ϵ^u, $\epsilon^d \sim \lambda$ (see the Table IV in the Appendix).

B. Masses and mixing from $\hat{Y}_K$

The matrix $\hat{Y}_0$ can be diagonalized by $U_0 = U_{23}U_{13}$, where the corresponding rotation angles equal $\tan\theta_{23} = \lambda + O(\lambda^3)$ and $\tan\theta_{13} = \lambda^2 + O(\lambda^4)$. After these rotations the 1-2 matrix becomes zero and therefore masses and mixing of the first and second generations are determined completely by the corrections to $\hat{Y}_0$. In fact, only 33 elements of matrices are nonzero and we will call this basis the "33" basis.

Formally one could work immediately in the "33" basis. In this basis however there is no guideline (apart from the experimental data) how corrections should be introduced. One can consider the matrix (13) as an ansatz for introduction of the corrections. It by itself leads to certain qualitative pattern of masses and mixing though quantitative predictions depend substantially on particular values of $|\epsilon|$'s within interval $(0 - \lambda)$. Furthermore the ansatz (13) has certain theoretical implications which we will outline in Section V. For a
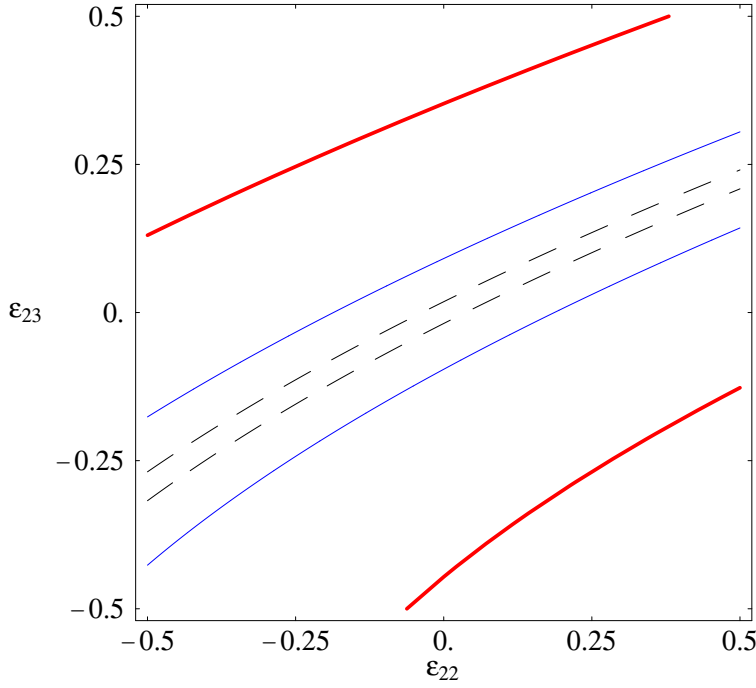
FIG. 1: The lines of constant ratio $m_\mu/m_\tau = 0.045$ (thick solid line), $m_s/m_b = 0.011$ (thin solid line), and $m_c/m_t = 0.0022$ (dashed line) in ϵ_{22}-ϵ_{23} plane at the GUT scale.

different ansatz where the dominant structure of the Yukawa matrices has a democratic form and related phenomenological considerations see [26, 27].

In what follows we find the parametric expressions for the observables in terms of λ and ϵ_{ij}^K. We discuss then restrictions on ϵ_{ij}^K and relations between them. The detailed study of ϵ_{ij}^K and their possible origins will be given elsewhere [28].

The complete mass matrix $\hat{Y}_K$ can be diagonalized with high accuracy by three successive rotations: $U = U_{23}U_{13}U_{12}$. The 2-3 rotation is determined by

$$\sin\theta_{23} \approx \lambda(1 + \epsilon_{23}), \tag{19}$$

and the 1-3 rotation—by

$$\sin\theta_{13} \approx \lambda^2(1 + \epsilon_{13}). \tag{20}$$

Here we omit the superscript for θ_{ij} and ϵ_{ij}, since these results apply to all fermions.

As a result of these two rotations we find the mass of the heaviest eigenstate

$$m_3 = 1 + \lambda^2 + O(\lambda^2\epsilon), \tag{21}$$

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and the matrix for the first and second generations:

$$\hat{m}_{12} = \lambda^2 \begin{pmatrix} \lambda^2(\epsilon_{11} - 2\epsilon_{13}) & \lambda(\epsilon_{12} - \epsilon_{13} - \epsilon_{23}) \\ \lambda(\epsilon_{12} - \epsilon_{13} - \epsilon_{23}) & \epsilon_{22} - 2\epsilon_{23} \end{pmatrix}, \tag{22}$$

where each matrix element is given in the lowest order in ϵ. Diagonalization of (22) gives

$$\tan 2\theta_{12} = 2\lambda \frac{(\epsilon_{12} - \epsilon_{13} - \epsilon_{23})}{\epsilon_{22} - 2\epsilon_{23} + O(\lambda^2\epsilon)} = \lambda \cdot r(\epsilon), \tag{23}$$

and masses of the lightest fermions

$$m_2 = \lambda^2(\epsilon_{22} - 2\epsilon_{23}) = \lambda^2 \cdot \epsilon \le \lambda^3, \tag{24a}$$

$$m_1 = \lambda^4 \left[\epsilon_{11} - 2\epsilon_{13} - \frac{(\epsilon_{12} - \epsilon_{13} - \epsilon_{23})^2}{\epsilon_{22} - 2\epsilon_{23}} \right] = \lambda^4 \cdot \epsilon. \tag{24b}$$

Here

$$r(\epsilon) \equiv \frac{\epsilon_1}{\epsilon_2} \tag{25}$$

where ϵ_1, ϵ_2 are functions of the order ϵ and parametrically $r(\epsilon) = O(1)$. However, strong cancellation can occur in ϵ_i. Also in some cases different terms in ϵ_i can sum up producing an enhancement. As a result, the ratio can be in rather wide range $r(\epsilon) \sim 10^{-1} - 10$.

Notice that the lightest mass is of the order $\lambda^4 \cdot O(\epsilon) \le \lambda^5 \sim 10^{-3}$ which gives correct order of magnitude for the down quarks and charged leptons.

The scenario predicts the following hierarchy of masses:

$$\frac{m_2}{m_3} = \lambda^2\epsilon, \quad \frac{m_1}{m_2} = \lambda^2 r(\epsilon), \quad \frac{m_1}{m_3} = \lambda^4\epsilon. \tag{26}$$

The experimental values of mass ratios, m_2^K/m_3^K and m_1^K/m_3^K, can be obtained provided that the combinations of ϵ in (26) take on the values given in the Table I. So, cancellation or

TABLE I: The values of combinations of ϵ in (26) that yield correct values of the mass ratios at the GUT scale. We take $\lambda = 0.26$.

	ϵ_u	ϵ_d	ϵ_l
m_2/m_3	0.032	0.16	0.66
m_1/m_3	0.0010	0.14	0.047

enhancement in the combinations of ϵ is needed which testifies that certain relations or/and

hierarchy between ϵ_{ij}^K exist. Random selection of parameters $|\epsilon_{ij}^K|$ in the intervals $(0 - \lambda)$ will not produce correct values of masses in most of the cases. The observables are very sensitive to choice of ϵ. It is this high sensitivity to ϵ that produces substantially different masses of up and down quarks and leptons.

Notice that according to (22) and (24b) both m_1 and 1-2 mixing will be enhanced if $\epsilon_{22} \approx 2\epsilon_{23}$.

The physical mixing matrix is a mismatch of the left rotations which diagonalize the mass matrices of the up and the down components of the weak doublets: $U = U(up)^\dagger U(down)$. Since the mass matrices of the up and down fermions are very similar, especially in 2-3 sector, they are diagonalized by rather similar rotations. In particular, the angles of up and down rotation have the same sign thus cancelling each other in the physical mixing matrix, so that $U \sim I$. This explains the smallness of the quark mixing angles. In contrast, due to the seesaw the neutrino rotation may flip the sign, so that the rotations in lepton sector will sum up leading to large mixing angles.

C. Quark mixing

The CKM matrix is given by

$$V_{CKM} = U_{12}^{u\ \dagger} U_{13}^{u\ \dagger} U_{23}^{u\ \dagger} U_{23}^{d} U_{13}^{d} U_{12}^{d}. \tag{27}$$

Using Eqs. (19), (20), and (23) we obtain the elements of the CKM matrix in the leading order in λ and ϵ:

$$V_{cb} \cong \lambda(\epsilon_{23}^d - \epsilon_{23}^u) = \lambda \cdot \epsilon, \tag{28a}$$

$$V_{ub} \cong \lambda^2 \left(\epsilon_{13}^d - \epsilon_{13}^u - \frac{(\epsilon_{23}^d - \epsilon_{23}^u)(\epsilon_{12}^u - \epsilon_{13}^u - \epsilon_{23}^u)}{(\epsilon_{22}^u - 2\epsilon_{23}^u)} \right) = \lambda^2 \cdot \epsilon, \tag{28b}$$

$$V_{us} \cong \lambda \left(\frac{(\epsilon_{12}^d - \epsilon_{13}^d - \epsilon_{23}^d)}{(\epsilon_{22}^d - 2\epsilon_{23}^d)} - \frac{(\epsilon_{12}^u - \epsilon_{13}^u - \epsilon_{23}^u)}{(\epsilon_{22}^u - 2\epsilon_{23}^u)} \right) = \lambda \cdot r(\epsilon). \tag{28c}$$

These elements have correct order of magnitude without any need for some special correlation between ϵ_{ij}^K. Indeed, for $\lambda = 0.26$, V_{cb} requires $\epsilon = 0.12 \approx 0.46\lambda$, V_{ub}: $\epsilon = 0.042 \approx 0.16\lambda$, and V_{us}: $r(\epsilon) \approx 0.86$.

The hierarchy of the quark mixings is naturally reproduced:

$$V_{us} \sim \lambda, \quad V_{cb}/V_{us} \sim \epsilon, \quad V_{ub}/V_{cb} \sim \lambda. \tag{29}$$

In Eqs. (A.1) and (A.2) of the Appendix we present two examples of corrections which reproduce all parameters of the quark sector. Notice that indeed, the inequalities ϵ_{ij}^u, $\epsilon_{ij}^d < \lambda$ are satisfied for all i, j. Both up and down matrices contain some elements of the order λ. Some corrections are much smaller than λ. Furthermore, two examples have different dominant structures (sets of matrix elements of the order λ). The detailed study of properties of ϵ_{ij} will be given elsewhere [28].

D. Lepton mixing: flipping the sign of rotation

In our approach an enhancement of the lepton mixing is a consequence of the seesaw mechanism. The seesaw produces two effects:

1. It flips the sign of rotation which diagonalizes the mass matrix of light neutrinos $\hat{m}_\nu$ with respect to the sign of the rotations which diagonalize the Dirac neutrino matrix $\hat{m}_D$ and charged lepton mass matrix. As a result, the rotations of the neutrinos and charged leptons sum up in the lepton mixing matrix;

2. It enhances moderately the mixing produced by the neutrino mass matrix.

Let us consider these effects for the 2-3 mixing explicitly. Diagonalizing the 2-3 submatrix of $\hat{m}_\nu$ we find

$$\tan 2\theta_{23}^\nu = 2\lambda \left[(1 + \epsilon_{23}^D) + \frac{(\epsilon_{23}^D - \epsilon_{23}^M)(\epsilon_{22}^D - 2\epsilon_{23}^D - \epsilon_{23}^{D\,2})}{\epsilon_{22}^M - 2\epsilon_{23}^M - 2\epsilon_{23}^D\epsilon_{23}^M + \epsilon_{23}^{D\,2} + \lambda^2 \cdot O(\epsilon)} \right]. \tag{30}$$

The first term in square brackets corresponds to diagonalization of the Dirac mass matrix; the second one is the effect of seesaw. An explanation of the magnitude of the 2-3 mixing requires the second term to be ~ -3. So that in combination with the first term it gives $\tan 2\theta_{23}^\nu \sim -4\lambda$.

Notice that the seesaw contribution is proportional to the difference of the off-diagonal (2-3) corrections and, approximately, the ratio of determinants of the Dirac and Majorana neutrino mass matrices. Since the determinants equal the corresponding mass hierarchies,

the enhancement of mixing requires much stronger hierarchy of the RN neutrino masses than hierarchy of the eigenvalues of the Dirac matrix.

This can be seen explicitly by considering the mass matrix of light neutrinos:

$$\hat{m}_\nu \sim \frac{1}{\epsilon_{22}^M - 2\epsilon_{23}^M - \epsilon_{23}^{M\,2}} \begin{pmatrix} A_{22}\lambda^2 & A_{23}\lambda \\ A_{23}\lambda & A_{33} \end{pmatrix}, \tag{31}$$

where $A_{ij} \equiv A_{ij}(\epsilon_{kl}^D, \epsilon_{kl}^M)$. We find explicitly that

$$A_{ij} = \epsilon_{22}^M - 2\epsilon_{23}^M + O(\epsilon_{ij}^2). \tag{32}$$

That is, the coefficients A_{ij} are all equal to each other in the lowest (first) order in ϵ_{ij}. Therefore to enhance the mixing and to flip the sign of rotation the terms of the order ϵ^2 in (32) should be important. Consequently,

$$\epsilon_{22}^M = 2\epsilon_{23}^M + O(\epsilon_{ij}^2) \tag{33}$$

and $A_{ij} = O(\epsilon_{ij}^2)$. The equality (33) means that the determinant of the Majorana matrix of the RH neutrino components is of the order $\lambda^2 \cdot O(\epsilon_{ij}^2)$ or smaller, and consequently, the RH neutrino masses have strong hierarchy:

$$\frac{M_2}{M_3} \sim \lambda^2 \cdot \epsilon^2 \leq \lambda^4, \tag{34}$$

whereas $m_2^D/m_3^D \sim \lambda^2\epsilon$. It is this difference of hierarchies which leads to the seesaw enhancement of the 2-3 mixing.

There are three different possibilities to realize the flip of the sign of the neutrino rotation:

1. Change the sign of the off-diagonal mass terms $(\hat{m}_\nu)_{23}$.

2. Change the sign of the diagonal mass term $(\hat{m}_\nu)_{33}$ (provided that $|(\hat{m}_\nu)_{33}| > |(\hat{m}_\nu)_{22}|$).

3. Enhance the 22 element, so that $(\hat{m}_\nu)_{22} > (\hat{m}_\nu)_{33}$.

In terms of Eq. (30) the sign of the second (seesaw) term can be changed in four different ways by appropriately changing the sign of the factors in its numerator and/or denominator. Numerically, we find this to happen in 5 % of cases for randomly generated coefficients ϵ_{22}^M, ϵ_{23}^M, ϵ_{22}^D and ϵ_{23}^D in the allowed range given in (14).

Summarizing, generically, the mass matrix of the left-handed (LH) neutrinos has the form (31) with moderately enhanced off-diagonal term: $|A_{23}/A_{33}| \sim 2 - 3$. The relative sign of

A_{23} and A_{33} is negative. In large region of parameter space A_{22} can be comparable with two other elements. That corresponds to summing up different (order ϵ^2) contributions, thus producing not too strong mass hierarchy.

E. Seesaw and the 1-2 neutrino sector

The mass matrix of the light neutrinos can be written as

$$\hat{m}_\nu = -U_L \hat{m}_D^{diag} V (\hat{M}_R^{diag})^{-1} V^T \hat{m}_D^{diag} U_L^T \quad , \tag{35}$$

where

$$V = U_R^\dagger U_M = U_{R12}^\dagger U_{R13}^\dagger U_{R23}^\dagger U_{M23} U_{M13} U_{M12}, \tag{36}$$

and U_R and U_M are the rotations of the RH neutrino components which diagonalize $\hat{m}_D$ and $\hat{M}_R$ correspondingly.

We find in the lowest order in λ and ϵ:

$$V \approx \begin{pmatrix} \cos\Delta_{12} & \sin\Delta_{12} & \cos\theta_{12}^R \sin\Delta_{13} - \sin\theta_{12}^R \sin\Delta_{23} \\ -\sin\Delta_{12} & \cos\Delta_{12} & \sin\Delta_{23} \\ -\cos\theta_{12}^M \sin\Delta_{13} + \sin\theta_{12}^M \sin\Delta_{23} & -\sin\Delta_{23} & 1 \end{pmatrix}, \tag{37}$$

where $\Delta_{ij} \equiv \theta_{ij}^M - \theta_{ij}^R$, and the angles θ_{ij} are determined in Eqs. (19), (20) and (23).

Due to equality $\epsilon_{22}^M \approx 2\epsilon_{23}^M$, according to (23) the angle θ_{12}^M can be near $\pi/4$, so that $\sin\Delta_{12} \sim 1$, $\cos\Delta_{12} \sim \lambda - 1$, $\sin\Delta_{23} = O(\lambda\epsilon)$, $\sin\Delta_{13} = O(\lambda^2\epsilon)$ and $\sin\theta_{12}^R = O(\lambda)$. Using these estimations we find

$$V \approx \begin{pmatrix} \lambda & 1 & \Delta_{13} - \Delta_{23}\lambda \\ -1 & \lambda & \Delta_{23} \\ \Delta_{23} & -\Delta_{23} & 1 \end{pmatrix}, \tag{38}$$

where $\Delta_{23} = (\epsilon_{23}^M - \epsilon_{23}^D)\lambda$ and $\Delta_{13} = (\epsilon_{13}^M - \epsilon_{13}^D)\lambda^2$. Taking the hierarchy of the mass eigenvalues as $\hat{M}_R^{diag} \sim (\epsilon \cdot \lambda^4, \epsilon^2 \cdot \lambda^2, 1)$ and $\hat{m}_D^{diag} \sim (\lambda^4, \epsilon \cdot \lambda^2, 1)$ we find from (38) an estimate of the light neutrinos mass matrix (before the LH rotations):

$$\hat{m}_D^{diag} V (\hat{M}_R^{diag})^{-1} V^T \hat{m}_D^{diag} \approx \begin{pmatrix} \lambda^6/\epsilon^2 & -\lambda^3 & \Delta_{23}\frac{\lambda}{\epsilon}(1 - \frac{\lambda}{\epsilon}) \\ -\lambda^3 & \epsilon & -\Delta_{23}/\lambda^2 \\ \Delta_{23}\frac{\lambda}{\epsilon}(1 - \frac{\lambda}{\epsilon}) & -\Delta_{23}/\lambda^2 & 1 \end{pmatrix} m_3. \tag{39}$$

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Note that Δ_{13} does not contribute in the leading order in λ. If we set $\Delta_{23} \approx \lambda^2$ the light neutrino mass matrix in the flavor basis with the LH rotating included takes the form

$$\hat{m}_\nu \approx \begin{pmatrix} \epsilon\lambda^2 & \epsilon\lambda & \lambda \\ \epsilon\lambda & \epsilon\lambda & 1 \\ \lambda & 1 & 1 \end{pmatrix} m_3, \tag{40}$$

where we show only the leading terms in both ϵ and λ. Notice that in the 12 element of (40) the combination ϵ should be enhanced to generate large 1-2 mixing. This will lead, after the 2-3 rotation, to the 1-3 term of the order λ^2 according to our general considerations.

IV. PHENOMENOLOGICAL CONSEQUENCES

The corrections ϵ_{ij}^K have been introduced in a certain way (13) and they are restricted to be small enough (14). This allows us to draw some qualitative consequences. Though exact predictions would require determination of ϵ_{ij}^K.

For illustration, in the Table IV we present two examples of the matrices of corrections. They correspond to two different realizations of the sign flip: the Example I(l) implements the inequality $(\hat{m}_\nu)_{22} > (\hat{m}_\nu)_{33}$ and in the Example II(l) the element $(\hat{m}_\nu)_{23}$ changes the sign. For simplicity we take $\epsilon_{13} = \epsilon_{11} = 0$. With these corrections the mass matrices reproduce precisely the lepton mixings, charged lepton masses and the neutrino mass squared differences.

The predictions from these two sets of matrices are given in the Table II where we present values of the lightest neutrino mass, the effective Majorana mass of the electron neutrino, the value of U_{e3} and masses of the RH neutrinos. Since the neutrino mass spectrum is hierarchical the radiative corrections are very small [29, 30].

A. 1-3 mixing

Generically for the 1-3 mixing we expect $U_{e3} \sim \lambda^2 \approx 0.07$. If $\epsilon_{13}^K = 0$ ($K = l, D, M$), we find

$$U_{e3} = \sin\theta_{13}^\nu - \sin\theta_{13}^l \cos\theta_{23} - \sqrt{\frac{m_e}{m_\mu}} \sin\theta_{23} + O(\lambda^4), \tag{41}$$

where θ_{13}^ν and θ_{13}^l are the angles of rotations which diagonalize the neutrino and charged lepton mass matrices, and $\theta_{23} \equiv \theta_{23}^\nu - \theta_{23}^l$. In (41) the last term is induced by simultaneous

12 and 23 rotations. Notice that in the sum each contribution is of the order λ^2 and the next order correction is very small. So, depending on sign of the angle and phase one may get substantial cancellation of the terms, and even $U_{e3} = 0$ can be achieved. If however the terms sum up we can get $U_{e3} \sim 0.2$ which corresponds to the present upper experimental bound.

We also refer to the results of the numerical analysis summarized in the Table II. (For details on numerical procedure see the Appendix.) In the examples considered in the Appendix, $|U_{e3}|^2$ is indeed of the order λ^4.

TABLE II: Majorana masses of the RH neutrinos, m_{ee}, m_1 and predicted value of $|U_{e3}|$. To extract the value of the RH neutrino masses we take $m_3 = 0.045\,\text{eV}$. M_i are given in GeV's, whereas the masses m_{ee} and m_1 are in eV's.

	M_1	M_2	M_3	m_{ee}	m_1	$|U_{e3}|^2$
Example I(l)	1.3×10^{10}	3.0×10^{10}	8.6×10^{14}	0.0006	0.002	0.008
Example II(l)	2.5×10^{8}	2.2×10^{11}	3.8×10^{14}	0.0007	0.004	0.001

B. The absolute scale of neutrino mass and m_{ee}

According to our general consideration in Section III E, the spectrum of light neutrinos is hierarchical, so that numerically m_3 and m_2 are determined by the mass squared differences measured in the atmospheric and solar neutrino experiments: $m_3 \approx \sqrt{\Delta m_{atm}^2} \cong 0.045\,\text{eV}$ and $m_2 \approx \sqrt{\Delta m_{sol}^2}$. Parametrically $m_2/m_3 = \lambda^2 \epsilon$ which implies that $\epsilon = 2.7$. The lightest mass can be found evaluating the determinants of the matrices in (12). Indeed, parametrically $\text{Det}\,(\hat{Y}_K) = \lambda^6 \cdot \epsilon_K^2$, where ϵ_K represents linear combinations of ϵ_{ij}^K coefficients, so that the determinant of the seesaw matrix $\text{Det}\,(\hat{m}_\nu) = \lambda^6 \epsilon_D^4/\epsilon_M^2$. Then, we have:

$$m_1 = \frac{\text{Det}\,(\hat{m}_\nu)}{m_2 m_3} = \lambda^4 \frac{\epsilon_D^4}{\epsilon \epsilon_M^2} m_3, \tag{42}$$

where $\epsilon_D = \epsilon_D(\epsilon_{ij}^D)$, $\epsilon_M = \epsilon_M(\epsilon_{ij}^M)$ and $\epsilon = 2.7$. In the examples presented in the Table IV the hierarchy of light masses is rather weak: $m_1/m_2 = 0.2 - 0.5$ which is partly related to strong hierarchy of the RH neutrino masses. For the lightest mass we get (see the Table II) typically

$$m_1 \sim (0.1 - 5) \times 10^{-3}\,\text{eV}. \tag{43}$$

Taking this into account we obtain $\epsilon_D^4/\epsilon_M^2 \sim (1 - 10^2)$ which can be used to estimate how singular $\hat{Y}_D$ and $\hat{Y}_M$ are with respect to each other.

The effective Majorana mass of the electron neutrino can be calculated immediately as the ee-element of the neutrino mass matrix in the flavor basis (40). Parametrically this gives $m_{ee} = \epsilon\lambda^2 m_3$, where ϵ stands for the linear combination of ϵ_{ij}'s.

Alternatively, we can use the neutrino masses and known neutrino mixing and present m_{ee} as the sum of contributions of mass eigenstates:

$$m_{ee} = \sum_i |U_{ei}|^2 m_i e^{i\phi_i} = m_{ee}(1) + m_{ee}(2) + m_{ee}(3). \tag{44}$$

In general, due to smallness of the 1-3 mixing the contribution of ν_3 is very small: $m_{ee}(3) = \sqrt{\Delta m_{atm}^2}\lambda^4 \sim 2 \times 10^{-4}\,\text{eV}$. The contribution of ν_2 is phenomenologically determined: $m_{ee}(2) = \sqrt{\Delta m_{sol}^2}\sin^2\theta_{sol} \sim (2 - 3) \times 10^{-3}\,\text{eV}$, and usually dominates. The contribution of ν_1 can be comparable with the previous one due to weak mass hierarchy and larger admixtures of ν_e. Furthermore, typically the masses and therefore contributions of ν_1 and ν_2 have an opposite sign cancelling each other in m_{ee}. For this reason the predictions for m_{ee} in the examples of the Table IV are small: $m_{ee} \sim 10^{-3}\,\text{eV}$.

If the Heidelberg-Moscow positive result [3, 4, 5, 6, 7] is confirmed, either our approach, at least in its present form, is not correct or another mechanism, different from the Majorana mass of the light neutrinos gives main contribution to the decay rate.

C. Leptogenesis

The corrections ϵ_{ij}^K are in general complex numbers and this is the source of CP violation in our approach.

Since $M_3 \gg M_2, M_1$, only two lighter RH neutrinos are relevant for leptogenesis and the lepton number asymmetry can be written as [31, 32, 33, 34, 35, 36]

$$\epsilon_L = \frac{1}{8\pi}\frac{M_1}{M_2}\frac{(h^\dagger h)_{12}^2}{(h^\dagger h)_{11}}. \tag{45}$$

Here h is the matrix of the Yukawa couplings in the basis where $\hat{M}_R$ is diagonal. Apparently,

$$h^\dagger h = V^T \frac{\hat{m}_D^{diag\ 2}}{v_1^2}V, \tag{46}$$

where V is determined in (38). Using estimations of the matrix elements of V (and assuming that the imaginary parts of these elements can be as large as the real ones) we find the asymmetry

$$\epsilon_L = \frac{\lambda^5}{8\pi} \sim 5 \times 10^{-5}. \tag{47}$$

Then the baryon to photon ratio is given by

$$\eta_B \sim 0.01 \epsilon_L k_1, \tag{48}$$

where k describes the washout of the produced lepton asymmetry due to weak deviation from the thermal equilibrium. The factor k depends on the effective mass parameter

$$\tilde{m}_1 = \frac{v_1^2 (h^\dagger h)_{11}}{M_1} \sim (0.1 - 1)\,\text{eV}. \tag{49}$$

For this value of the effective mass we get $k_1 \sim 10^{-3}$, and therefore $\eta_B \sim 5 \times 10^{-10}$ in agreement with the observed value.

Notice that the key difference of our scenario from the analysis in [37] is that the lightest eigenvalue of the neutrino Dirac matrix is much larger than the up quark mass m_u; in the Example I(1): $m_{1D} \sim 300\,\text{MeV}$. Also, the left rotations are not negligible here.

D. Lepton number violating effects

In the SUSY context one expects observable flavor violating decays, like $\mu \to e\gamma$, due to slepton mixing related to the neutrino mixing. The approximate formula for the $\mu \to e\gamma$ branching ratio, which has the most stringent experimental limit, reads

$$BR(\mu \to e\gamma) \simeq \frac{\alpha^3}{G_F^2} \frac{|(\delta m_{\tilde{L}}^2)_{21}|^2}{m_s^8} \tan^2 \beta, \tag{50}$$

where m_s stands for the effective mass of the superparticles, and $\delta m_{\tilde{L}}^2$ represents the off-diagonal corrections to the slepton mass matrix. They appear due to the renormalization group running between the scale where universality conditions on SUSY breaking parameters are imposed, which we take to be the GUT scale M_{GUT}, and scale where the RH neutrinos decouple from the theory.

We find

$$(\delta m_{\tilde{L}}^2)_{ij} \approx \frac{3m_0^2 + A_0^2}{8\pi^2} (V_L)_{i3} (\hat{Y}_D^{diag})_{33} (V_L^\dagger)_{3j} \ln \frac{M_{GUT}}{M_3}, \tag{51}$$

where the matrix $V_L = U_l^\dagger U_L$ represents the mismatch in the LH rotations that diagonalize $\hat{Y}_l$ and $\hat{Y}_D$. The relevant coefficients $(V_L)_{13}$ and $(V_L)_{23}$ for the $\mu \to e\gamma$ process are proportional to $\lambda^2 \cdot O(\epsilon)(\le \lambda^3)$ and $\lambda \cdot O(\epsilon)(\le \lambda^2)$ respectively. Though, the exact values depend on combinations of ϵ, we expect the product $(V_L)_{13}(V_L)_{23}$ to be close to $V_{ub}V_{cb} \le \lambda^5$. (See also the form of V in Eq. (37) and the discussion on the mixing in the quark sector.)

Rather precise approximation for the effective mass m_s is given by [38]

$$m_s^8 \simeq 0.5 m_0^2 M_{1/2}^2 (m_0^2 + 0.6 M_{1/2}^2)^2, \tag{52}$$

where m_0 is the typical slepton mass and $M_{1/2}$ is the gaugino mass. Taking for simplicity, $m_0 = M_{1/2} \equiv m$ and value $(V_L)_{13}(V_L)_{23} = \lambda^5$ we obtain

$$BR(\mu \to e\gamma) \simeq 1.8 \times 10^{-9} \left(\frac{m}{100\,\text{GeV}} \right)^{-4}, \tag{53}$$

where $\lambda = 0.26$, $\tan\beta = 55.9$, $(\hat{Y}_D^{diag})_{33} \simeq 0.7$, $M_{GUT}/M_3 = 100$ and $A_0 = 0$ were used. In the case of an exact quark-lepton symmetry: $(V_L)_{13}(V_L)_{23} = V_{ub}V_{cb}$ we find

$$BR(\mu \to e\gamma) \simeq 1.1 \times 10^{-11} \left(\frac{m}{100\,\text{GeV}} \right)^{-4}. \tag{54}$$

According to Eqs. (53) and (54) for $m \simeq (300 - 400)\,\text{GeV}$ we expect $BR(\mu \to e\gamma) \simeq 10^{-13} - 10^{-11}$. This interval is close to the current experimental limit of $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [39] and clearly within reach of the MEG experiment at PSI [40] which will have a sensitivity down to $BR(\mu \to e\gamma) \le 5 \times 10^{-14}$.

The results of an exact numerical running[44] of slepton mass matrix are also in an excellent agreement with the approximations presented in this section.

Finally, we note that the majority of the SUSY GUT models yields significantly larger value for the product $(V_L)_{13}(V_L)_{23}$ than what is generated in our approach. This puts them in precarious position with respect to the experimental constrains on lepton flavor violating processes. Namely, they typically yield $(V_L)_{13}(V_L)_{23} \sim 10^{-2}$–$10^{-1}$ (see for example [42] and references therein) which makes them violate the experimental bounds even for low values of $\tan\beta$ (~ 5). On the other hand generically we obtain $(V_L)_{13}(V_L)_{23} \sim 10^{-4} - 10^{-3}$, which can be traced back to the ansatz (10). This rather large suppression more than compensates the enhancement of $\mu \to e\gamma$ branching ratio that originates from the large value of $\tan\beta$. The suppression brings our prediction for $\mu \to e\gamma$ branching ratio close to but below the current experimental limit.

V. DISCUSSION AND IMPLICATIONS

1). There are two different approaches to the theory of fermion masses. One possibility (widely explored in the literature) is to build up the theory immediately on the basis of observables—the mixing and mass ratios—considering them as fundamental parameters. In this case the quark-lepton symmetry is *strongly* broken at least by masses of the first and second generations. In a number of models this is described by introduction of different charges for the leptons and quarks. Another approach is when the quark-lepton symmetry is *weakly* broken. In this case, the observables appear as diagonalization of the nearly equal mass matrices. They are determined by small corrections to the dominant structure equal for quarks and leptons.

2). The main feature of our approach is the nearly singular matrices $\hat{Y}_K$. This allows us, using small perturbations, to generate strong difference of the mass hierarchies of quarks and leptons and simultaneously enhance the lepton mixing. The lepton mixing (due to the seesaw mechanism) is unstable with respect to perturbation of the RH mass matrix which appears in the denominator of the expression for the light neutrino masses. The perturbations of M_R influence strongly the mass hierarchy of the RH neutrinos and therefore (via the seesaw enhancement) the mixing of light neutrinos.

3). The matrix $\hat{Y}_0$ (10) can be obtained in the model with U(1) family symmetry in the context of the Froggatt-Nielsen [43] (F-N) mechanism. According to this mechanism the Yukawa couplings are generated by the operators

$$a_{ij} f_{iL}^c f_{jL} \left(\frac{\sigma}{M_F} \right)^{q_i+q_j} H_k, \tag{55}$$

where f_i are fermion components, a_{ij} are dimensionless constants of order 1, and σ is the scalar field—singlet of the SM gauge symmetry group with a $U(1)_F$ charge of -1. H_k ($k = 1, 2$) are the Higgs doublets of the MSSM, M_F corresponds to mass scale at which the non-renormalizable operators describing interactions of σ with fermion fields are generated and q_i ($i = 1, 2, 3$) are the $U(1)_F$ charges of the family i.

After σ develops the VEV $\langle \sigma \rangle$ the following Yukawa couplings are generated:

$$(\hat{Y})_{ij} = a_{ij} \lambda^{q_i+q_j}, \quad \lambda = \frac{\langle \sigma \rangle}{M_F}. \tag{56}$$

If $a_{ij} = a_0 = 1$, the matrix $\hat{Y} = \hat{Y}_0$ is singular. Furthermore, prescribing the $U(1)_F$ charges 0, 1, and 2 for the third, second, and first family we reproduce the required structure of matrix $\hat{Y}_0$ (10). Corrections to Y_0 can be generated by deviations of a_{ij} from universality:

$$a_{ij} = a_0(1 + \epsilon_{ij}). \tag{57}$$

In general, the required singular matrix can be represented as the product:

$$\hat{Y}_0 = W \times W^T, \quad W^T \equiv (a_1, a_2, a_3). \tag{58}$$

In turn, such a structure can appear as a result of interaction of the light fermion fields (f_1, f_2, f_3) with a single heavy field F. Let us consider the following mass terms:

$$\bar{F}_L \sum_{i=1}^{3} \mu_i f_{iR} + \bar{F}_R \sum_{i=1}^{3} \mu_i f_{iL} + h.c. \tag{59}$$

with $\mu_i < M$ and the Dirac mass terms formed by f_{iL} and f_{iR} are forbidden by some symmetry. Then after decoupling of F we get for the light masses

$$m_{ij} = \frac{\mu_i \mu_j}{M} \tag{60}$$

with required properties. Notice however, that this mechanism cannot be applied immediately to top quark since $m_t \sim \mu_i|_{max}$.

4). To reproduce observables we still need small deviations of coefficients a_{ij} from 1. This may come from the F-N mechanism itself as it is indicated in (57) or from new physics at some higher scale as an additional contribution to mass matrices. The correction matrix is of the order $\lambda^2 - \lambda^3 \sim (1 - 3) \times 10^{-2}$. So, if the flavor symmetry is realized at the GUT scale the correction matrix can be related to some physics at the string scale.

5). The case of unstable matrices reproduces to some extent a situation of anarchy: small perturbations of the otherwise symmetric pattern lead to significant difference in the observables.

Selecting a_i in (58) one can further "optimize" the structure of the dominant singular matrix to reduce spread of the the corrections ϵ_{ij}^K, to diminish their absolute values or to get certain relations [28].

6). To explain the observed masses and mixing certain relations between the correction parameters ϵ should be satisfied and some of them should be in narrow ranges. These relations should be used to construct the theory of ϵ. Random selection of values of $|\epsilon|$'s in the intervals $0 - \lambda$ produces typically incorrect values of the observables.

VI. CONCLUSIONS

We have elaborated an approach in which no *ad hoc* symmetry for the neutrino sector is introduced. The difference of parameters in the quark and the lepton sectors arises essentially from the seesaw mechanism as well as from "instability" of the mass matrices. The difference of the mass hierarchies follows from small perturbations of the singular matrices. Singularity can be a consequence of certain family symmetry.

The explanation of features of observables is reduced to large extent to explanation of perturbations (corrections). Particular values of ϵ's are needed. Still our proposal opens alternative approach to explain the data. Furthermore, in this approach one gets

1. correct hierarchy of the quark mixings;

2. hierarchical mass spectrum of light neutrinos;

3. 1-3 mixing of the order λ^2;

4. small effective Majorana mass of the electron neutrino: $m_{ee} \leq 10^{-2} \, \text{eV}$;

5. in general, deviation of the 2-3 mixing from maximal;

6. generic prediction is a strong mass hierarchy of the second and third RH neutrinos which is of the order λ^4.

Perturbations of the singular matrix introduced in the form (13) with $|\epsilon_{ij}| \leq \lambda = 0.26$ allow us to reproduce all available experimental results. Even in parametric form the approach leads to correct qualitative pattern of masses and mixings though quantitative description of the data requires precise determination of $|\epsilon_{ij}|$ within the interval $0 - \lambda$. Let us summarize the information on ϵ we have obtained:

- We have shown that the data can be well described for all $\epsilon \leq 0.26$.

- There is rather strict relation (33) required by the enhancement of the 2-3 leptonic mixing. The same relation also gives enhancement of the 1-2 mixing.

- ϵ_{23} for the Dirac mass matrices are determined by the mass ratios for the second and third generations.

- ϵ_{22} elements correlate with ϵ_{23} and they are restricted by the 2-3 CKM mixing in the quark sector.

- Values of ϵ_{11} are practically irrelevant.

- There are rather complicated relations between other parameters (they also include parameters of the 2-3 sector) which follow from masses of first generation, the 1-2 leptonic mixing and CKM mixing. These relations do not restrict a given parameter once other parameters are allowed to change in the intervals $|0 - \lambda|$.

- The description of all available data leaves substantial freedom of variations of these parameters (ϵ_{12}, ϵ_{13}). So one can impose on them additional conditions motivated by theoretical context (zeros, equalities, *etc.*) [28].

VII. ACKNOWLEDGMENTS

APPENDIX

We present the numerical results for ϵ_{ij}^K corrections. Input parameters, the masses and mixings of the matter fields, at the GUT scale used for the numerical fit, are given in the Table III. We assume the MSSM particle content below the GUT scale and determine $\tan\beta$ requiring the unification of b and t Yukawa couplings.

In Eqs. (A.1) and (A.2) we present two examples of correction matrices of quarks which yield exact agreement with the experimental input in the Table III.

TABLE III: Experimental values of the quark and charged lepton masses and relevant CKM angles extrapolated to the GUT scale. Three-loop QCD and one-loop QED renormalization group equations are used in running up to m_t. Further extrapolation from m_t to $M_{GUT} = 2.3 \times 10^{16}$ GeV is done using the two-loop MSSM beta functions taking all SUSY particles to be degenerate at m_t and assuming $\tan\beta = 55.9$. Masses are given in GeV.

m_u	m_c	m_t	m_d	m_s	m_b	m_e	m_μ	m_τ	$|V_{us}|$	$|V_{ub}|$	$|V_{cb}|$
0.000558	0.264	121	0.00137	0.0239	2.16	0.000530	0.110	2.45	0.222	0.00284	0.0320

Example I(q):

$$\epsilon^u \cong \begin{pmatrix} 0 & 0.0683 & -0.0103 \\ 0.0683 & 0.144 & 0.0526 \\ -0.0103 & 0.0527 & 0 \end{pmatrix} \quad \epsilon^d \cong \begin{pmatrix} 0 & 0.0387 & -0.163 \\ 0.0387 & -0.00386 & -0.0821 \\ -0.163 & -0.0821 & 0 \end{pmatrix} \quad (\text{A.1})$$

Example II(q):

$$\epsilon^u \cong \begin{pmatrix} 0 & 0.00811 & -0.0100 \\ 0.00811 & 0.0200 & -0.00782 \\ -0.0100 & -0.00782 & 0 \end{pmatrix} \quad \epsilon^d \cong \begin{pmatrix} 0 & -0.0112 & -0.160 \\ -0.0112 & -0.110 & -0.141 \\ -0.160 & -0.141 & 0 \end{pmatrix} \quad (\text{A.2})$$

The coefficients (y_u, y_d) in the examples I(q) and II(q) are $(0.645, 0.655)$ and $(0.650, 0.659)$ respectively. This difference can also be accounted as λ^2 correction to (33) elements. The parameter λ is always 0.26.

We next specify two examples in the lepton sector in the Table IV. In the spirit of the simplest $SO(10)$ model we set $y_D = y_u \cong 0.645$ for both cases. Our fit yields $y_l = 0.753$ in the Example I(l) and $y_l = 0.754$ in the Example II(l).

TABLE IV: Corrections ϵ_{ij}^K in the lepton sector $(K = l, D, M)$ for two different cases which realize different scenarios for the flip of the sign of rotations. The fit is performed assuming $m_2/m_3 = 0.187$ at the GUT scale. We also require $\tan^2 \theta_{sol} = 0.4$ and $\sin^2 2\theta_{atm} = 0.95$.

Example I(l) $((\hat{m}_\nu)_{33} < (\hat{m}_\nu)_{22})$	Example II(l) $((\hat{m}_\nu)_{23} < 0, \ (\hat{m}_\nu)_{33} > 0)$

$$\epsilon^l \cong \begin{pmatrix} 0 & -0.171 & -0.036 \\ -0.171 & 0.254 & -0.268 \\ -0.036 & -0.268 & 0 \end{pmatrix} \qquad \epsilon^l \cong \begin{pmatrix} 0 & -0.093 & 0.006 \\ -0.093 & 0.262 & -0.262 \\ 0.006 & -0.262 & 0 \end{pmatrix}$$

$$\epsilon^D \cong \begin{pmatrix} 0 & 0.233 & 0 \\ 0.233 & 0.104 & 0.042 \\ 0 & 0.042 & 0 \end{pmatrix} \qquad \epsilon^D \cong \begin{pmatrix} 0 & 0.213 & 0 \\ 0.213 & -0.200 & 0.098 \\ 0 & 0.098 & 0 \end{pmatrix}$$

$$\epsilon^M \cong \begin{pmatrix} 0 & 0.0065 & 0 \\ 0.0065 & 0.0098 & 0.005 \\ 0 & 0.005 & 0 \end{pmatrix} \qquad \epsilon^M \cong \begin{pmatrix} 0 & 0.130 & 0 \\ 0.130 & 0.264 & 0.129 \\ 0 & 0.129 & 0 \end{pmatrix}$$

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